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The effect of correlation between random variables on reliability-based design optimization for mechanical components

Murat Mayda [1*]

[1]Department of Mechanical Engineering, Karamanoglu Mehmetbey University, Karaman,Turkey

A B S T R A C T : The correlation between design variables needs to be taken into consideration in the RBDO of mechanical components because an optimum design found without consideration of the correlation could be a weak or over design, or unreliable design when the correlation exists among the system variables. To clearly illustrate the significant effect of the correlation of random variables on the efficiency and effectiveness of the RBDO process, in this work, a case study, which is the RBDO of a cantilever beam with correlated random variables, is carried out using Monte Carlo Simulations (MCS). From the results of the simulations, it can be possible to conclude that an increase in correlation coefficient significantly increases the difficulty of finding the optimum point in RBDO problems. However, the influence of correlation coefficient on the RBDO performance can vary depending on the complexity of a design problem. The main critical point is that correlation coefficient must be accurately determined prior to the RBDO process; otherwise, it can be indispensable to negatively affect the RBDO performance and results.

Keywords: Correlation, random variable, design optimization, mechanical components.

1. Introduction

Reliability-based design optimization (RBDO) deals with searching for an optimum design point, subject to a desired reliability or a limit of probability of failure for a system or product. The existing random variables in a system can be correlated or uncorrelated. In most of the RBDO problems solved in the literature, the random variables have been assumed to be uncorrelated. On the other hand, the effect of correlation between design variables on the RBDO performance and results needs to be investigated because an optimum design found without consideration of the correlation could be a weak or over design, or unreliable design when the correlation exists among the system variables. Accordingly, there have been several research attempts. For example, Noh, et al. [1], using the Nataf transformation, carried out the RBDO process in the consideration of correlations in the design variables. Lee, et al. [2] conducted a RBDO with confidence level on a design problem with correlated and a lack of statistical information. Li and Lu [3] implemented a sensitivity analysis for design problems with correlated inputs. Zhang, et al. [4] investigated the impact of correlated random variables on the production performance by introducing confidence level to random variables. Paulson, et al. [5] used arbitrary polynomial chaos (aPC) for propagation of correlated random variables. However, these works focus on the impact of the correlation on the RBDO results and performance, there still needs to conduct a research to

* Corresponding author e-mail: mmayda@kmu.edu.tr
Tel.: +91 0000000000

clearly illustrate the significant effect of the correlation of random variables on the efficiency and effectiveness of the RBDO process. For this purpose, in this work, a case study, which is the RBDO of a cantilever beam with correlated random variables, is carried out to clearly show how the correlation affect the performance and results of the RBDO.

The rest of this paper is organized as follows: in Section 2, the statistical terms and the definition of design optimization under uncertainty are briefly explained. In Section 3, the case study, which is the RBDO of a cantilever beam with correlated random variables, is carried out. In Section 4, a conclusion about the results of the investigation is drawn.

2. Definition of design optimization under uncertainty

In this section, first a short explanation of correlation coefficient and covariance is presented; afterwards, the definition of design optimization under uncertainty is included. Correlation coefficient (ρ_{XY}) varies between -1 and 1, referring to the degree of positive or negative correlation between the associated variables. The value of -1 indicates a strong negative correlation, the value of 1 means a strong positive correlation, and 0 indicates there is no correlation between these variables.

If two random variables (X,Y) are correlated, the covariance, σ_{XY} , can be defined as [6]:

$$\sigma_{XY} = Cov(X,Y) = \rho_{XY}\sigma_X\sigma_Y \tag{1}$$

where ρ_{XY} is correlation coefficient, σ_X and σ_Y are the standard deviations of X and Y, respectively.

As for the design optimization formulation under uncertainty, the definition of the RBDO can be expressed as follows [6]:

$$\left.\begin{array}{l} Min\ \ f(X,\mu_B) \\ subject\,to: \\ \qquad P(L_m(X,B)) \geq R_m,\ \ m=1,...,s \\ \quad or\ \ P(L_m(X,B)) \leq P_f \\ \qquad X_i^L \leq X \leq X_i^U,\ i=1,...,r \end{array}\right\} \tag{2}$$

where f denotes the objective function. B represents the vector of random design variables. μ_B is the mean value of a random variable. L_m represents a limit-state function. P(·) is the probability related to the limit-state function. Rm and Pf refer to the target reliability and the probability of failure of the system, respectively.

3. A case study: RBDO of a cantilever beam with correlated random variables

A cantilever beam is considered to illustrate the how the correlation between design variables affects the performance of the RBDO process. The beam is exposed to a force of 2000 N, and assumed to be 0.2 m long (Figure 1). Also the height (h) and width (b) are shown in this figure.

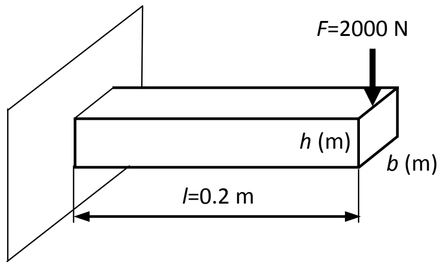


Figure 1. The dimensions of the cantilever beam

In this design optimization, the objective is to minimize the volume of the beam subjected to a probability of 0.01 that the maximum deformation exceeds 0.002 m, which is defined by,

$$\left.\begin{array}{l} Min\ \ f(l,b,h) \\ subject\,to: \\ \qquad P(\dfrac{Fl^3}{3EI}-0.002)\geq 0.01 \\ \qquad 0.015\leq h\leq 1.5,\ 0.02\leq b\leq 1.2 \end{array}\right\} \tag{3}$$

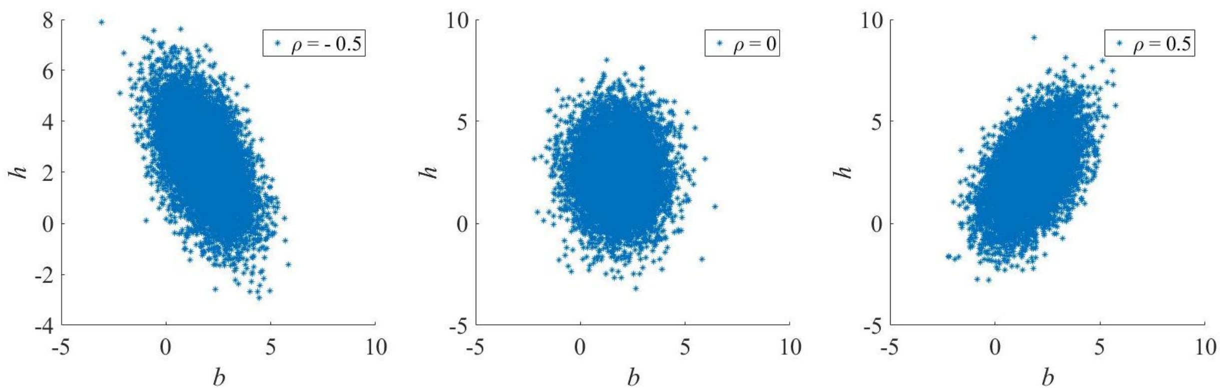
The case of being correlated or random variables of the variables used in the example are presented in Table 1. F and l are assumed to be deterministic variables and not be correlated. h and b are assumed to be correlated Normal random variables.

Table 1. The statistical information of the variables used in the example

Variable		Correlated	Type of distribution	Coefficient of variation
F	Deterministic	-	-	-
l	Deterministic	-	-	-
h	Random	(h, b)	Normal	0.3
b	Random	(b, h)	Normal	0.3

The correlation coefficients between b and h, which were obtained by MCS with 10,000 realizations, are shown in Figure 2. In this analysis, the ρ is considered within the range from -1 to 1 with the increment of 0.1.

After implementing the RBDO process by using MCS with 50,000 realizations under the conditions of correlation aforementioned, the best volume values (f(X)) for each condition that is subject to ρ were achieved as seen in Figure 3. From the figure, it can be clearly stated that an increase in ρ increases the difficulty of finding the optimum point in RBDO problems. On the other hand, the influences of ρ on the RBDO performance can vary depending on the complexity of a design problem. Herein the main critical point is that ρ must be accurately determined prior to RBDO process; otherwise, it can significantly affect the RBDO performance and results.



a) ρ = -0.5 b) ρ = 0 c) ρ = 0.5

Figure 2. Negative, zero and positive correlations between b and h.

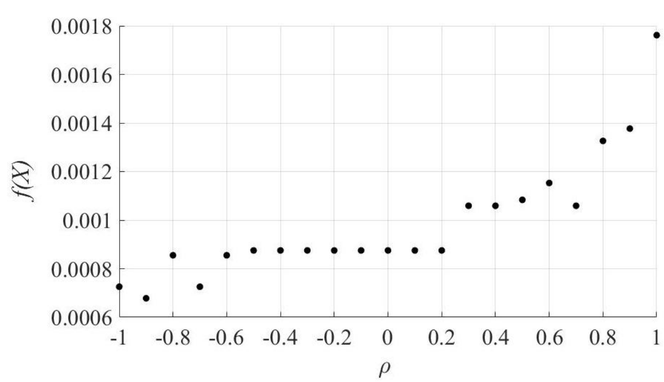


Figure 3. The optimum value f(X) versus ρ.

Conclusion

In this work, the effect of the correlation coefficient (ρ) on the performance of the RBDO process was investigated on the design of a cantilever beam by using MCS. From the results of the simulations, it was concluded that an increase in correlation coefficient increases the difficulty of finding the optimum point in RBDO problems. However, the influence of correlation coefficient on the RBDO performance can vary depending on the complexity of a design problem. The main critical point is that correlation coefficient must be accurately determined prior to RBDO process; otherwise, it can negatively affect the RBDO performance and results. In future works, based on the procedure proposed in this work, several complex RBDO examples can be considered, and thus, it can be possible to draw more useful deductions about the effects of correlation coefficient on the RBDO process.

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